Chapter 2: Standard Deviation and Correlation

**Explanations of the answers for the self-test questions, Chapter 2**

1. The standard deviation can be appropriately used with data measured at any level of measurement, quantitative, rank, or qualitative. True or False?
   - **Answer = False.** The standard deviation is a measure of deviation from the mean. In qualitative and rank order data no mean can be appropriately computed, so no measure of deviation from the mean would be appropriate.

2. The ACT scores of the entering class at a particular college have a mean of 21 and a SD = 4. From this you can conclude that about 95% of the students in the entering class would have scores that ranged between 17 and 25. True or False?
   - **Answer = False.** About 95% would fall between 2 SD above and 2 SD below the mean, that is, between 13 and 29. The range of 17 to 25 is 1 SD above and below, so it would contain about 2/3 of the students.

3. Consider two groups of 300 students and their scores on the same test. Group 1's scores have a mean of 70 and a SD of 5. Group 2's scores have a mean of 65 and a SD of 20. Which of the following is true of these two sets of scores?
   a. The individual student with the highest score on the test will be a member of Group 1.
   b. There will be almost no overlap in the scores of the students in the two groups.
   c. A score of 80 equates to a higher z-score in Group 1 than in Group 2.
   d. If the distribution is approximately normal, about two-thirds of the scores in Group 1 will fall between 70 and 75.
   - **Answer = c.** A score of 80 in Group 1 is 2 SDs above the mean or \( z = 2 \). A score of 80 in Group 2 is \( 4 \) of a SD above the mean (15/20); \( z = .75 \).

4. Concerning standard scores it is accurate to say
   a. A standard score cannot be calculated unless the distribution of scores is normal.
   b. A standard score on a norm-referenced test such as the SAT measures one's rank rather than one's absolute level of achievement.
   c. The z-score cannot be converted into another of standard score unless the distribution of scores is normal.
   d. All of the above are accurate.
   - **Answer = b.** A standard score indicates how many standard deviations above or below the mean a particular score is located. The mean might be low (or high) and indicate a low (or high) absolute level of achievement. Options a and c are incorrect because calculating standard scores does not require the distribution to be normal.

5. You compute a correlation between two variables: \( r = .50, \ p = .009 \). About this finding, it is accurate to say
   a. By most standards, the result is not statistically significant.
   b. The relationship between the two variables is inverse.
   c. Knowing one variable enables you to predict or explain 25% of the variance in the other variable.
   d. One variable cannot be causally related to the other since they are correlated at such a low level of statistical significance.
• Answer = c. Using scores on one variable to predict those on another is done with the squared correlation coefficient or \( r^2 \). Answers a and d are incorrect because a p-value of .009 is smaller than nearly every conventional cutoff for statistical significance; this is the sort of result most researchers would describe as “highly significant.” Answer b is wrong because the correlation of .50 (with no minus sign) is positive or direct, not negative or inverse.

6. A negative correlation indicates that the relationship between two variables is unfavorable or disadvantageous. True or False?
• Answer = False. A negative correlation between two sets of scores means only that they tend to move in opposite directions, such as the correlation between low levels of air pollution and high lung capacity.

7. A correlation of 2.34 between an IV and a DV means that the DV is over 2 times as large as the IV. True or False?
• Answer = False. A correlation of 2.34 is meaningless since the highest possible correlation is 1.0.

8. When using Pearson r correlation
a. it is important to check for linearity, since \( r \) will overestimate a non-linear relationship
b. one obtains evidence only about pairs of variables
c. non-linear relationships between variables are better measured by the \( r \)-squared statistic rather than by the simple \( r \)
d. all of the above are true.
• Answer = b. You can’t compute a Pearson \( r \) with one or more than two variables, only with two. If you have more than two (as in the pairs of test scores in the following table), you can study them only two at a time. Answers a and c are incorrect because the Pearson \( r \) underestimates non-linear relations; squaring it doesn’t help with that problem.

<table>
<thead>
<tr>
<th>Correlations of 4 test scores (for questions 9-12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
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<tr>
<td>--------</td>
</tr>
<tr>
<td>Test 1</td>
</tr>
<tr>
<td>Test 2</td>
</tr>
<tr>
<td>Test 3</td>
</tr>
<tr>
<td>Test 4</td>
</tr>
</tbody>
</table>

*\( p < .05 \),  **\( p < .01 \)

The following 4 questions (9 through 12) refer to the correlation matrix, above.

9. Because the table is incomplete, it is not possible to examine the correlations between all pairs of tests. True or False?
• Answer = False. All necessary information is included. The blank spaces are made up first of the correlation of each test with itself, which is always 1.0. The rest of the blanks are redundant information. For example, the correlation of Test 1 with Test 2 (.37) is the same as the correlation of Test 2 with Test 1; it need not be in the table twice.

10. The correlation between Test 1 and Test 3 is significant at the .05 level. True or False?
• Answer = False. The correlation is .03; its level of significance is not indicated (no asterisk).
11. The strongest correlation in the table is between Test 1 and Test 4. True or False?
   • Answer = True. The strength of a correlation is measured by its absolute size, whether it is positive or negative. The negative correlation between Test 1 and Test 4 is the strongest in the table. The variance in Test 1 predicts the variance in Test 4 better than it predicts the variance in Test 2 and Test 3.

12. All of the correlations in the table are statistically significant. True or False?
   • Answer = False. Significance in this table is indicated by asterisks, which refer to the significance levels in the footnote (*p < .05 and **p < .01). Two of the correlations in the table have no asterisk.

13. Concerning the relation between the size of a correlation, the size of the sample on which it was computed, and its level of statistical significance, it is accurate to say
   a. the larger the sample, the easier it is to find a correlation that is statistically significant
   b. all else equal, the larger the correlation the larger the p-value
   c. in two samples of the same size, the larger the correlation the less likely it is to be statistically significant
   d. all of the above are accurate
   • Answer = a. In very large samples even a quite small correlation (e.g., $r = .15$) could be statistically significant; in small samples, even quite large correlations (e.g., $r = .75$) might not be statistically significant. Answers b and c are incorrect; the larger the correlation, the smaller the p-value, or, in other terms, the greater the statistical significance--all else equal.

• Additional discussion questions and exercises for students

Collect two measurements from each member of the class: height in inches and hand span in centimeters (span is the distance between the tip of the thumb and the tip of the little finger when the hand is spread as widely as it will go without forcing it). One of the nice things about using this kind of data for a correlation exercise is that it is the kind of measurement used by Galton and Pearson in the 1880s and 1890s to illustrate the first published correlations.

Use the directions in Table 2.1 (page 21 of the text) to compute the mean and standard deviation for height and span. Then use that information and the directions in Table 2.2 (page 26) to compute the correlation between height and span. In a large class, take a sample. No more than 20 sets of measurements are needed, and calculating a correlation with more than that many is tedious in the extreme. It is useful to do the calculations by hand so that students can see how a correlation develops out of the data. Then address the following questions:
   a. What would the correlation between height and span be if all students had identical spans?
   b. How would the size of the correlation be affected if all the measurements were taken by a medical professional using high-quality instruments? In other terms, does more precise measurement influence the size of a correlation? If so, how?
   c. Could you use the information from this group of students to make any generalizations about the likely correlation between height and span in a broader population?
d. Are height and span causally related? Are they both caused by another variable or variables?
e. Change two of the measurements. Make the span for the shortest student 30 and the span for the tallest student 10. What effect will these changes have? Recompute, either by hand or using a statistical package, study the results and discuss what they mean for interpreting correlations.
f. How many students and their measurements would you need to determine whether there was a difference in the height-span correlation between males and females? In which group would you guess that the correlation was stronger? Why?

- Examination questions for Chapter 2 (an asterisk indicates the correct answer)

1. Which of the following correlations shows the strongest relationship?
   a. -.20
   b. +.65
   c. -.05
   d. -.85*
   e. +.70

2. If the Pearson $r$ correlation equals 1.0, the means of the two variables will necessarily be identical. True or *False?

3. A correlation of 1.0 indicates a perfect positive relationship between two variables; by contrast, a correlation of -.1.0 would indicate that there is no relationship whatsoever between them. True or False*?

4. A correlation of 1.96 is significant at the .05 level if the distribution of the data is normal. True or False*?

5. A correlation matrix showing the bivariate correlations among 5 variables will show _____ distinct (non-redundant) correlations.
   a. 25
   b. 5
   c. 10*
   d. the number cannot be determined from the information given.

6. If a researcher reports a correlation of 1.96, this means that
   a. the relationship is nearly two times as strong
   b. the coefficient is most likely statistically significant
   c. *a typographical or other error has been made
   d. the effect size of the correlation is 1.96.

7. If a researcher reports a correlation of .01, this means that
   a. the probability that a result of this size is due to chance alone is 1%
   b. the coefficient is statistically significant
   c. 1% of the variance in one variable can be explained by the variance in the other
   d. *the association between the correlated variables is small.
8. Two variables are considered to be "independent" if the correlation of scores on the variables turns out to be close to 1.0. True or False*?

9. Because the decision about how to label a Pearson r is somewhat subjective and related to context, it is possible for two reasonable researchers to use different labels (e.g., "high," "moderate," or "low") even if their correlation coefficients were exactly the same size. True* or False?

**Questions 10 – 13 pertain to Figure 2.1, following question 13**

10. Concerning the two scatter diagrams in Figure 2.1, it is accurate to say: the correlation between x1 and y1 is stronger than that between x2 and y2. True* or False?

11. Concerning the two scatter diagrams in Figure 1, it is accurate to say: even though the correlation between x2 and y2 is positive and statistically significant [it is: \( r = .50, p = .011 \)], the scatter diagram shows that a correlation coefficient does a poor job of describing the relation between the variables. *True or False?

12. Concerning the two scatter diagrams in Figure 1, it is accurate to say: the correlation between x1 and x2 is about \( r = .25 \). True or False*?

13. The x1-y1 relationship is homoscedastic (equality of variances) while the x2-y2 relationship is heteroscedastic (inequality of variances). True* or False?
Figure 2.1. Scatter diagrams, for questions 10 – 13
14. If the mean income for nurses working in a particular city was $35,000 per year and the standard deviation was $5,000, then the $z$-score for a nurse earning $40,000 would be 1.0. *True or False?*

15. Because the standard deviation is a measure of deviation from the mean, it cannot be calculated when all or most of the scores in a distribution are at or very near the mean. True or *False?*

16. Unlike the standard deviation, the variance is a measure of how scores are associated; it is not a measure of their relationship to a distribution’s mean. True or *False?*

17. Which of the following scores indicates the highest rank?
   a. GRE of 600
   b. IQ of 120
   c. 75th percentile
   d. $z$-score of 2.0*  

18. Which of the following correlation coefficients would be most useful for predicting scores on one variable with scores on another variable? In other words, which shows the strongest statistical association between variables?
   a. $R^2 = .55$
   b. $r^2 = .64$*  
   c. $r = .70$
   d. $R = .75$

19. Because one is a measure of difference from the mean, while the other is a measure of association, the $z$-score and the Pearson $r$ are based on totally different information about scores on variables. True or *False?*

20. Consider the following sets of scores for Var. 1 and Var. 2. Would the correlation between the two variables be
   a. positive*  
   b. negative
   c. impossible to determine

<table>
<thead>
<tr>
<th>Var. 1</th>
<th>Var. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5</td>
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<tr>
<td>-3</td>
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<tr>
<td>7</td>
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<td>-2</td>
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<tr>
<td>-5</td>
<td>-3</td>
</tr>
</tbody>
</table>

21. A correlation of .44 would be more statistically significant (have a lower $p$-value) in a sample of 300 than in a sample of 30. True* or False?
### Table 2.1. Correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>5.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Income</td>
<td>--</td>
<td>--</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Age</td>
<td>.67*</td>
<td>--</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Savings rate</td>
<td>.73**</td>
<td>.79**</td>
<td>--</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Vacation spending</td>
<td>.05</td>
<td>.24*</td>
<td>.13</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>5. Technology spending</td>
<td>.35*</td>
<td>-.44*</td>
<td>-.29*</td>
<td>.31*</td>
<td>--</td>
</tr>
</tbody>
</table>

*p < .05, **p < .01

**Questions 22-25 refer to Table 2.1, above**

22. Older people tended to earn more. *True or False?

23. Younger people tended to spend less on technology. *True or *False?

24. The correlation between income and vacation spending is significant at the .05 level. *True or *False?

25. The correlation between income and technology spending is stronger than the correlation between age and technology spending. *True or *False?

26. A correlation of .75 will always be more practically important than a correlation of .25. *True or *False?

27. The Pearson $r$ will always underestimate a curvilinear relationship. *True or False?

28. If two variables are strongly and significantly (statistically) correlated, this tells you
   a. nothing about any potential causal relationship between them
   b. *that they are more likely to be causally related than if they were not correlated
   c. that they are definitely causally related, but we cannot determine which is the cause and which is the effect
   d. that there is more likely to be a positive than a negative correlation between them.