Chapter 2 Selected Worked Exercises

1. Using truth tables, prove the identities of Section 2.1.2. For de Morgan's law: $\neg (P /Q) = (\neg P \wedge \neg Q)$

P	Q	¬P	¬Q	P ∨ Q	¬ (P ∨ Q)	¬ P ∧ ¬ Q	=
T	T	F	F	T	F	F	T
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	F	T	T	T

2. A new operator *exclusive-or* may be defined by the following truth table:

P	Q	P exor Q
T	T	F
T	F	T
F	T	T
F	F	F

Create a propositional calculus expression using only $\wedge, \vee$ and $\neg$ that is equivalent to P exor Q. Prove their equivalence using truth tables.

P	Q	$P \vee Q$	$P \wedge Q$	$\neg (P \wedge Q)$	$(P \vee Q) \wedge \neg(P \wedge Q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

3. The logical operator $\Leftrightarrow$ is read if and only if. $P \Leftrightarrow Q$ is defined as being equivalent to $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$.

Based on this definition, show logical equivalence of $P \Leftrightarrow Q$ and $P \vee Q \Rightarrow P \wedge Q$:

(a) By using truth tables.

P	Q	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$	$P \vee Q$	$P \wedge Q$	$(P \vee Q) \Rightarrow (P \wedge Q)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	T	F	F	T

5. (a) Prove that modus ponens is sound for propositional calculus. Hint: use truth tables to enumerate all possible interpretations, then show that wherever the premises are true, the first line in the truth table below, the conclusion is also true.

P	Q	$P \Rightarrow Q$	$P \wedge (P \Rightarrow Q)$	$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

6. Attempt to unify the following pairs of expressions. Either show their most general unifiers or explain why they will not unify.

(a) p(X,Y). and p(a,Z). Unifies with { a/X, Y/Z}.

(b) p(X,X). and p(a,b). Fails to unify since both a and b can not be substituted for X.

(c) ancestor(X,Y). and ancestor(bill, father(bill)). Unifies {bill/X, father(bill)/Y}.

(d) ancestor(X,father(X)). and ancestor(david, george). Unifies only if george is the father(david). Function evaluation should be added to the unification algorithm.

(e) q(X). and $\neg$ q(a). Will unify if provision is made for $\neg$ in the unification algorithm.

(f) p(X,a,Y). and p(Z,Z,b). Unifies with {a/X, a/Z, b/Y}.

10. Jane Doe has four dependents, a steady income of 30,000.00, and 15,000.00 in her savings account. Add the appropriate predicates describing her situation to the general

investment advisor of the example in Section 2.4 and perform the unifications and inferences needed to determine her suggested investment.

dependents(4) means **minincome(20000)** and by using rule 5, **savings(inadequate)**. Using rule 1, we can determine that **investment(savings)**.

11. Write a set of logical predicates that will perform simple automobile diagnostics (e.g., if the engine won't turn over and the lights won't come on, then the battery is bad). Don't try to be too elaborate, but cover the cases of bad battery, out of gas, bad spark plugs and bad starter motor.

    We have created several such knowledge bases in the supplementary programming materials.

12. The following story is quoted from N. Wirth's "Algorithms + data structures = programs" (Wirth 1976).

    I married a widow (let's call her W) who has a grown-up daughter (call her D). My father (F), who visited us quite often, fell in love with my step-daughter and married her. Hence my father became my son-in-law and my step-daughter became my mother. Some months later, my wife gave birth to a son (S1), who became the brother-in-law of my father, as well as my uncle. The wife of my father, that is, my step-daughter, also had a son (S2).

    Using predicate calculus, create a set of expressions that represent the situation in the above story. Add expressions defining basic family relationships such as the definition of father-in-law and use modus ponens on this system to prove the conclusion that "I am my own grandfather." This works fine with a little violation of the accepted semantics for family relationships!